LAB-2
2D LINEAR SYSTEMS

INTRODUCTION

In this exercise, we will focus on the implementation of 2D linear systems. In J-DSP, 2D linear systems can be specified in two ways: First, by **2D Filter** block that implements the 2D convolution sum (Fig. 1(a)), and second, by **2D FastFilter** block that implements FFT-based fast convolution using the overlap-add method (Fig. 1(b)).

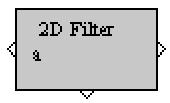


Fig. 1(a) **2D Filter** block

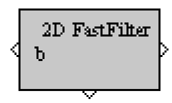


Fig. 1(b) **2D FastFilter** block

Fig. 2 shows the 2D J-DSP editor. In fig. 2, the 2D blocks that are placed in the vertical line on the left hand side are called permanent blocks and are shown in green color. The blocks placed in the horizontal line are shown in yellow color and can be changed by selecting one of the options given in the drop-down menu on the top left corner of the screen. It is always recommended to use any 2D block separately before it is connected with others and to read the [Help] screen of every new block you use.

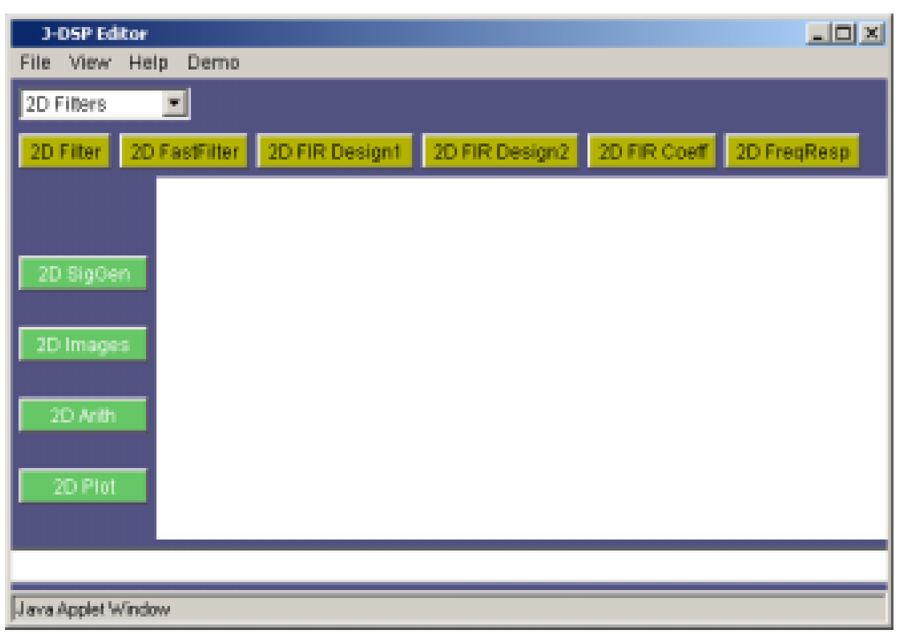


Fig.2 2D J-DSP.

Two-dimensional linear finite impulse response (FIR) digital filters are linear, shift-invariant two-dimensional systems, whose regions of supports are finite. The direct implementation of a 2D FIR filter is very easy and can be performed using a 2D convolution sum. Any 2D FIR filter can also be implemented by using a 2D FFT algorithm as shown below in Equation (1). The overlap-add and overlap-save methods are FFT-based methods and can be used to perform the filtering operation. In some cases, these methods reduce the number of arithmetic operations significantly, relative to realization by direct convolution.

$$y(n_1,n_2) = h(n_1,n_2) ** x(n_1,n_2) \leftrightarrow Y(\omega_1,\omega_2) = H(\omega_1,\omega_2) X(\omega_1,\omega_2) \qquad (1)$$

** is for 2D convolution.

In Equation (1), $y(n_1,n_2)$ is the output of the system.

$x(n_1,n_2)$ is the input to the system and $h(n_1,n_2)$ is the impulse response of the system.

The corresponding capital letters show the 2D signals in frequency-domain.

Note: Give reasons for all the answers to the questions, and where necessary, give appropriate J-DSP plots (Time/Frequency-domain) in order to justify your answers. You may need to use other 2D blocks as well.

Problem 2.1

Given the following 2D sequences,

$$x(n_1,n_2) = \begin{cases} 0.5 & , \ (\ n_1 = 0, \ \ n_2 = 0) \\ 0.25 & , \ (|\,n_1\,| = 1, \ \ n_2 = 0) \\ 0.25 & , \ (\ n_1 = 0, \ |\,n_2\,| = 1) \\ 0.125 & , \ (|\,n_1\,| = 1, \ |\,n_2\,| = 1) \end{cases}$$

$$h(n_1,n_2) = \begin{cases} 1.0 & , \ (n_1 \geq n_2) \\ 0 & , \ n_1, n_2 < 0 \\ 0 & , \ n_1, n_2 > 3 \end{cases}$$

Evaluate $x(n_1,n_2) ** h(n_1 - 1, n_2 + 1)$, where (**) denotes 2D convolution.

a) What is the region of support (ROS) of the output signal?
b) Is the output of this 2D convolution separable?
c) What are the benefits of designing separable linear shift-invariant (LSI) systems?

Your system should look as shown in Fig. 3

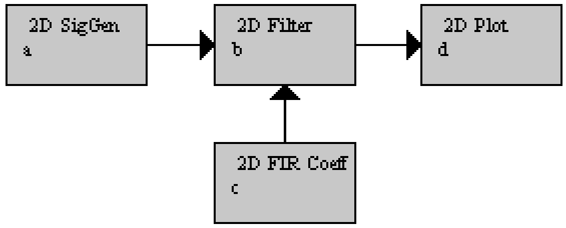


Fig.3

Problem 2.2

Use 2D J-DSP to compute the 2D convolution of the two 2D signals using DFT method. Choose (4*4)-point DFT size. The two signals to be convolved are as follows:

$$x(n_1,n_2) = \delta(n_1,n_2)+0.3\delta(n_1-1,n_2)-\delta(n_1,n_2-1)-0.5\delta(n_1-1,n_2-1)+\delta(n_1,n_2-2)+\delta(n_1-2,n_2)-\delta(n_1-3,n_2) + 2\delta(n_1,n_2-3)-\delta(n_1-2,n_2-2)$$

$y(n_1,n_2) = 1.0 \quad ; 0 \le n_1, n_2 \le 2$

Note: You have to use FFT and IFFT blocks in order to do FFT-based fast convolution.

Problem 2.2.1

Use **2D Filter** block in order to convolve the same two signals as in **problem 2.2**.

a) Are the results of the two systems equal?
b) Which system results in a circular convolution?
c) What changes need to be made in order to avoid the circular convolution?

Problem 2.3

Now connect the 2D blocks as shown in the Fig. 4.

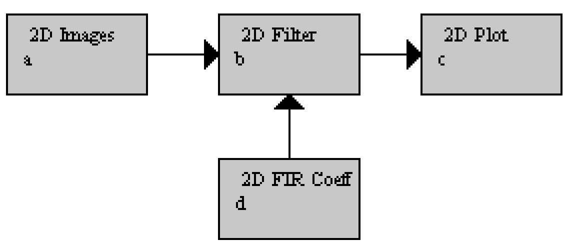


Fig. 4

In the **2D FIR Coeff** block, set the values of the impulse response of the system as follows:

$h(n_1,n_2) = 0.1111 \quad ; -1 \le n_1, n_2 \le 1$

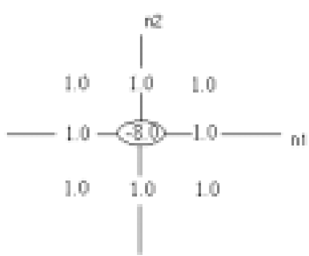
In the **2D Images** block, select “Cameraman” as an input to the system.

a) What happens to the image, when it’s passed through the filter?
b) What kind of filtering effect has the system on the input image?
   i. LPF
   ii. HPF
   iii. BPF
   iv. BSF

Problem 2.4

Now change the impulse response of the system as follows:

$h(n_1,n_2)$ =



The encircled value is at the origin (0,0).

a) What happens to the image, when it’s passed through the filter?
b) What kind of filtering effect has the system on the input image?
   i. LPF
   ii. HPF
   iii. BPF
   iv. BSF

Problem 2.4.1

Use the same impulse response as in **problem 2.4** and change the value at: h(0,0) = -9, -8, -7

a) Did you observe any change at the output-image when changing the value from –9 to –7?

Hint: Take a closer look at the coefficients of the impulse response of the 2D FIR filters.

Problem 2.5

In the **2D Images** block, select “Lena” as an input to the system. Insert Guassian noise of density=0.08.

Use the following 2D convolution kernels in the **2D FIR Coeff** block.

1. $h(n_1,n_2) = 0.1111$; $-1 \le n_1,\ n_2 \le 1$
2. $h(n_1,n_2) = 0.0204$; $-3 \le n_1,\ n_2 \le 3$

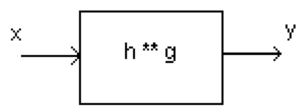
a) What happens when the kernel is too small?
b) What happens when the kernel is too large?

Note: In order to see the noisy image, you need to connect the **2D Image** block with the **2D Plot** block directly or select the "impulse" as an impulse response in the **2D FIR Coeff** block to see the noisy image. At this point, you cannot see the noisy image in the **2D Image** block itself.

Problem 2.6

Answer the following general questions.
Note: No plots are required. State answers clearly.

a) Give equivalent block diagram of the system shown below (without convolving h with g).



b) Two 2D sequences, each of which is 5*6 points in extent, are circularly convolved using (8*8)-point DFT. Which samples of the (8*8)-point output array are identical to the samples of the linear convolution of the two input arrays, and which are different.

c) What happens when a blurred image is subtracted from the original image?

d) Below are the three impulse responses of the 2D casual systems. The lower left corner is at the origin.i.e., h(0,0).

0	-1	0
-1	5	-1
0	-1	0

(1)

1	-2	1
-2	5	-2
1	-2	1

(2)

-1/7	-2/7	-1/7
-2/7	19/7	-2/7
-1/7	-2/7	-1/7

(3)

i. What type of filters/systems are these?

ii. One characteristic of all these filters/systems is that the sum of all the amplitudes of each impulse response is one. Does this have anything to do with the intensity of the input signal/image?

Hint: Take a closer look at the numerical values of the frequency response of the 2D FIR filters. Frequency response is the Fourier transform of the impulse response of the system.

e) If you need to reduce the random noise in a signal, which impulse response would you use, the one in **problem 2.3** or the one in **problem 2.4**? If you need to sharpen edges, which one would you use?